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A new method for solving linear Fredholm integral equation of the first kind

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Abstract

This paper presents an effcient method for approximation solution of Fredholm integral equation of first kind (FIEFK) based on combine of two methods, Galarkin and Artificial Bee Colony (ABC) in two step. In the first step, we applied Galarkin method and obtain the residual function of the FIEFK. Then, in the second step, we have minimized residual function in the before step by ABC algorithm. Finally, result of the mentioned method is satisfactory and acceptable with respected to other methods.

Keywords: Fredholm integral equation, Cumulative algorithms, Artificial bee colony

1 Introduction

Integral equations play important role in the mathematics, science and engineering. Also, the FIEFK problem is one of the integral equations that operator forms of it is same as follows:

$$Y = \lambda \mathrm{KU} \qquad (1)$$

Which Y and k are known function, λ is a constant parameter and U is the unknown function to be determined. Hence, we know the analytic solution for this kind of equation doesn't exist in general form. Then, use numerical methods to determine the approximate solution. We must pay attention to the resistance of the method, when approximating the solution by numerical methods. If the operator k is reversible and its reverse is bounded then, the equation will be well-posed. Otherwise it will be ill-posed. In this case, may be the solution of it is not exist and/or handling inverse problems lead to a lot of difficulties. Several methods have been used to handle the (FIEFK), for example the classical method of successive,Variation iteration method, Homotopy analysis method, Legendre wavelets method and artificial neural networks (NNs) see [25, 3, 2, 9, 18, 19, 14, 4, 10, 11]. Although, some of these techniques have excellent convergence characteristics and are widely used in the industry. But, determination the approximate solution of an equation leads to a system of linear equations that, coefficient matrix of it is ill-posed (The condition matrix number is very large). Hence, may be obtain non exact solutions because, the small change to the problem make very large change to the obtained answers. So, they are developed with some theoretical assumptions and fail with systems having non-smooth, unbounded and non-differentiable functions and others constraints. Therefore, to overcome the above-mentioned drawbacks we have proposed an efficient iterative method, which can obtain with combination Galerkin method and ABC algorithm (i.e., bees colony algorithm [12]) for first time. So that, simulation results obtained of the proposed method respect to others methods is remarkable and very well and comparative studies same as [1, 5, 20, 21, 22, 23] have shown that ABC is faster and more efficient than other heuristic algorithms in benchmark problems.

2 Preliminaries

2.1 Galerkin method

Linear Fredholm integral equation of the first kind, its form is as follows.

$$y(x) = \lambda \int_a^b k(x,t)u(t)dt, \;\; x \in [a,b]. \tag{2}$$

Where $y(x)$ and $k(x,t)$ are known function and $u(x)$ is the unknown function to be determined. In order to, solve FIEFK in equation (2), the classic Galerkin method consists of seeking and approximate solution of form.

$$u_n(x) = \sum_{i=1}^{n} a_i \varphi_i(x) \tag{3}$$

And puting (3) in to (2), we find that

$$r_n(x) = \lambda \int_a^b k(x,t)u(t)dt - y(x), \quad x \in [a,b] \tag{4}$$

Where $r_n(x)$ is the residual such that $r_n(x) = 0$ for $u_n(x) = u(x)$. However, for compute $a_{i,}i = 1,2, \dots, n,$ we must have

$$< r_n, \varphi_i(x) >= 0, \qquad i = 1,2, \dots, n \qquad (5)$$

That mean, inner product of them was be zero. Therefore, for determining the approximate solution of an equation we have a system of linear equations. So that if the coefficient matrix of it is ill-posed then, maybe obtain non exact solutions. Because the small change to the problem make big changes to the obtained answers.

2.2 ARTIFICIAL BEE COLONY ALGORITHM

Bio-inspired algorithms are amongst the most powerful algorithms for the optimization problems [1, 6,7, 8, 17, 28, 29], especially for the non-linear hard problems (NLHP). Artificial Bee Colony method is one of this algorithms that produced by Dervis Karaboga in 2005 [15, 16, 24]. So that, three bee categories exist in the artificial bee colony: the employed bees, onlookers and scouts. For each food source, there is one employed bee. The scouts are the colony's foragers and every bee colony has them. The scouts are described by scant search costs. Scouts also have a scant average in source quality. Sometimes, the scouts can detect randomly the best food sources. In the ABC algorithm, the food source places show a feasible solution to the optimization problem and amount of nectar of related foods shows the quality of the related solution. The number of solutions and employed bees and onlooker bees are alike. Each solution xi, (i = 1, 2, ..., SN) is a D dimensional vector, where SN denotes the size of population. An employed bee gives a correction of the situation (solution) in her mind depending on the native information and tests the nectar amount of the new source (new solution). While the amount of nectar of the new solution is higher than the previous solution, the bee keeps the new position and forgets the previous position [13, 26]. Otherwise the bee puts place of the previous solution in her memory and forget new one. If the nectar amount of a food source was low, then the bee forgets this source and scouts replace it by new food source. The role employed bee of a low nectar amount food source will be changed to a scout. There are three main control parameters that are used in the ABC: the number of food sources and employed and onlooker bees (SN) that are equal, the value

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of limit, the maximum cycle number ([13, 15, 16, 27]). Short pseudo-code of the ABC algorithm is given in the Algorithm 1.

1. Initial food sources are produced for all employed bees.
2. **Repeat** UNTIL (requirements are met)
   (a) Each employed bee goes to a food source in her memory and determines a neighbor source, then evaluates its nectar amount and dances in the hive.
   (b) Each onlooker watches the dance of employed bees and chooses one of their sources depending on the dances, and then goes to that source. After choosing a neighbor around that, she evaluates its nectar amount.
   (c) Abandoned food sources are determined and are replaced with the new food sources discovered by scouts item. The best food source found so far is registered.

Algorithm 1: Artificial Bee Colony Algorithm.

3 Solution of Fredholm integral equations

According to subsection 2.1, we know the form of FIEFK is same as follows.

$$y(x) = \lambda \int_a^b k(x,t)u(t)dt, x \in [a,b). \quad (6)$$

Now, we obtain an approximation solution of equation (6) by following

Steps

Step 1: We define approximation solution of FIEFK as follows

$$u_n(x) = \sum_{i=1}^{n} a_i \varphi_i(x) \quad (7)$$

Also, with substituting (7) in to (6), we get

$$r_n(x) = \lambda \int_a^b k(x,t)u(t)dt - y(x), \quad x \in [a,b)$$

or equality, we have

$$= \lambda \int_a^b k(x,t) \sum_{i=1}^{n} a_i \varphi_i(x)\, dt - y(x), \quad x \in [a,b)$$

$$= \lambda \sum_{i=1}^{n} a_i \int_a^b k(x,t)\varphi_i(t)dt - y(x), \quad x \in [a,b)$$

$$= \lambda \sum_{i=1}^{n} a_i\, f_i(x) - y(x), \quad x \in [a,b)$$

In which that,

$$f_i(x) = \int_a^b k(x,t)\varphi_i(t)dt.$$

In the second step, our goal is to compute $a_i, i = 1,2, \dots, n$ such that $r_n(x) = 0$. Therefore, to obtain fine results in any case, we must be minimize $|r_n(x)|$.

Step 2. In this part, we use of ABC algorithm for obtain optimal solution of the following problem

$$Minimize \quad r(x,z) = |r_n(x)| = \left|\lambda \sum_{i=1}^{n} a_i\, f_i(x) - y(x)\right| \qquad (8)$$

with respect to all variables $a_{1,} a_2, \dots, a_n$. Because, for get good results, we must be minimize residual in each process. Then, we can say, pseudo-code of proposed method for solving FIEFK is same as Algorithm 2.

Input: FIEFK problem, MCN = maximum cycle number

SN = the number of food sources. $z_{j,min}$ = the lower limits of the jth optimization variable, $z_{j,max}$ =the upper limits of the jth optimization variable

Output:z^*: best optimal solution of FIEFK problem,

Initialize: the SN population of solutions $z_i = (a_1{}^i, a_2{}^i, \dots, a_n{}^i)$, $i = 1,2,\dots,SN$,

$$z_{ij} = z_{j,min} + \text{ran}[0,1](z_{j,max} - z_{j,min}). \qquad (9)$$

$i \in \{1,2,\dots,SN\}$ and $j \in \{1,2,\dots,n\}$.

Evaluate: the fitness value for each employed bee by using the following

equation: $$fit(x, z_i) = fit(x)_i = \frac{1}{(1+r(x,\ z_i))}. \qquad (10)$$

In which that $r(x,\ z_i)$ is the value of the functional residual.

cycle = 1,

Generate new solutions v_i for the employed bees as following equation

$$v_{ij} = z_{ij} + \emptyset_{ij}(z_{ij} - z_{kj}) \qquad (11)$$

where$k \in \{1,2,\dots,SN\}$ and $j \in \{1,2,\dots,n\}$ are randomly chosen indices, z_{kj} is a randomly

chosen solution different from z_{ij}, and v_{ij} is the new solution (food source).

Also, $\emptyset_{ij}$ denotes a random number in the interval [-1; 1].

Apply the greedy selection process for z_i and v_i

Calculate the probability value pi corresponding to the solution z_i by using.

$$\mathrm{p_i} = \frac{\mathrm{fit(x)_i}}{\sum_{\mathrm{i=1}}^{\mathrm{SN}} \mathrm{fit(x)_i}} \qquad (12)$$

Produce the new solutions v_i for the onlooker bees from the old solutions z_i selected depending on $\mathrm{p_i}$ in above equation and evaluate their fitness values.

Apply the greedy selection process between the old solution z_i and new solution v_i.

Determine the abandoned solution for the scout, if exists, and replace it with a new randomly produced solution. Memorize the best solution found so far.

$$cycle = cycle + 1$$

if $cycle < MCN$ and $r(x,\ z_i) > \epsilon$ **then**

Generate new solutions $v_i$for the employed bees by following equation and evaluate their fitness values.

$$v_{ij} = z_{ij} + \emptyset_{ij}(z_{ij} - z_{kj}) \qquad (13)$$

else

| STOP

end

end

Algorithm 2: Proposed method

6

ABC algorithm can be one. ABC algorithm begins to work by randomly producing solutions. The initial solutions are produced for employed bees by using Equation (1).

4 Numerical example

In this section to illustrate the effectiveness of the proposed method we run the algorithm ABC on numerical examples. Solve this equation by using previous methods may be lead to a lot of difficulties. By applying the Algorithm ABC on numerical examples we can remove all the difficulties in previous methods.

Example1: Consider the equation,

$$\frac{\text{sinx} - \text{xcosx}}{\text{x}^2} = \int_0^1 \sin(xt)\, u(t)dt. \qquad (14)$$

The exact solution for this problem is $u(x) = x$. By applying the algorithm to equation (11) with n = 2, '$\varphi_{i1}(x) = 1, \varphi_2(x) = x$, the error is 0, because z = (0, 1). Figure 1 compares the exact solution with approximate solution obtain with proposed method.

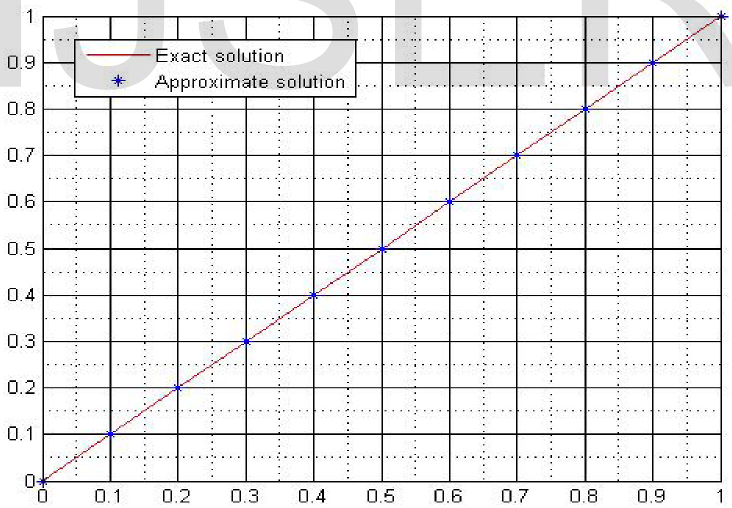

Fig1.Results for Example 1

Example 2: Consider the equation,

$$\frac{\pi}{2} sinx = \int_0^{\pi} \sin(x - t)\, u(t)dt \qquad (15)$$

The exact solution for this problem is $u(x) = cosx$. This problem is ill-posed. Because that condition number is very large. Therefore, sometimes the other methods cannot solve this example. But, by using proposed method with n=3 and $\varphi_i(x) = \sum_{i=1}^{N} a_i x^{2i-2}$ the maximum error is 0.015 and solutions are acceptable. So that, Table 1 show the numerical results and Figure 2 compares the exact solution with approximate solution of Example 2.

x_i	Exact solution	Approximate solution
0	1	1
0.1	0.99500416	0.99526889
0.2	0.98006657	0.98111024
0.3	0.95533648	0.95762809
0.4	0.92106099	0.92499584
0.5	0.87758256	0.88345625
0.6	0.82533560	0.83332144
0.7	0.76484218	0.77497289
0.8	0.69670670	0.70886144
0.9	0.62160996	0.63550729
1	054030230	0.55550000

Table1: Numerical results for Example 2

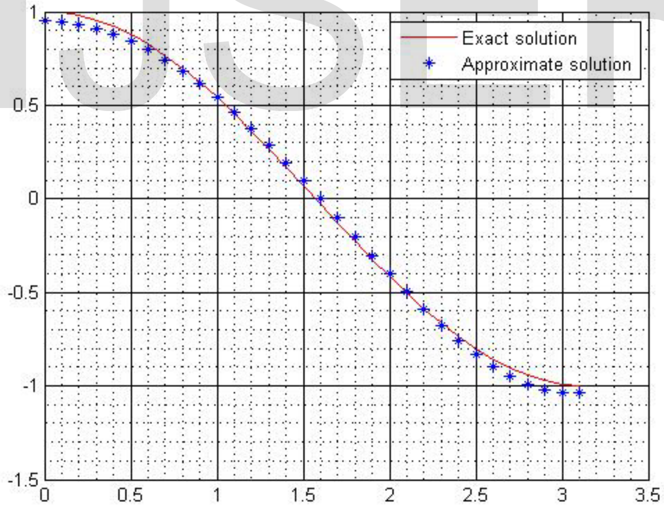

Fig2.Results for Example 2

Example 3. Consider the equation,

$$\frac{e^{x+1}}{x+1} = \int_0^1 e^{xt} u(t)dt, \qquad x \in [0,1) \qquad (16)$$

The exact solution for this problem is u(x) = e^x. By using the algorithm presented in this paper, an acceptable approximate solution is obtained that the maximum error is 0:0005 and to be seen in the Table 2. As, the

Figure3 compares the exact solution with approximate solution of Example 3 for n=6.

x_i	Exact solution	Approximate solution
0	1	1
0.1	1.105170918	1.104822894
0.2	1.221402758	1.220772448
0.3	1.349858808	1.349183722
0.4	1.491824698	1.491413376
0.5	1.648721271	1.648843750
0.6	1.822118800	1.822886944
0.7	2.013752707	2.014498889
0.8	2.225540928	2.226633472
0.9	2.459603111	2.459346526
1	2.718281828	2.714700000

Table2: Numerical results for Example3

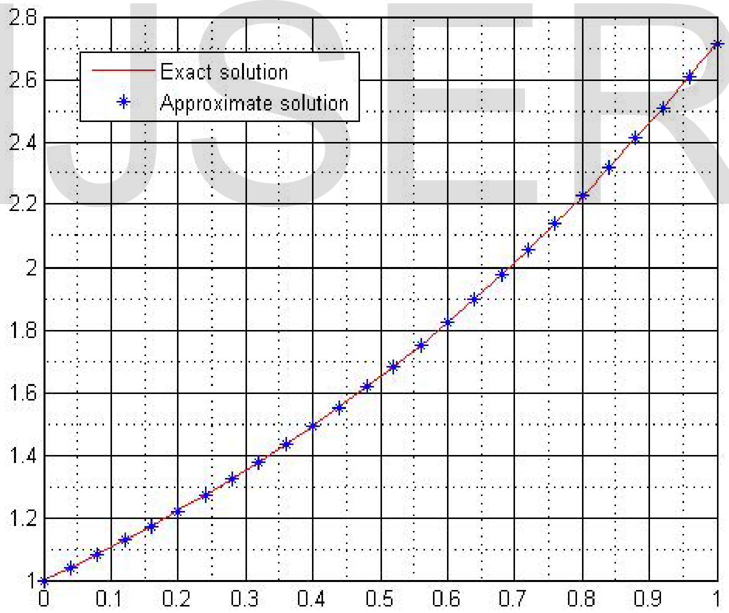

Fig3.Results for Example 3

5 Conclusion

Consequently, in this paper, we proposed an efficient method for approximation solution of Fredholm integral equation of first kind based on hybrid artificial bee colony. So that, in theory this method we have not need to assumptions and fail to deal with systems having smooth, bounded, continuous and differentiable functions same as other methods. Finally, the analyzed examples illustrated the ability and reliability of the present approach. The obtained solutions, in comparison with exact solutions admit a remarkable accuracy.

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